So, You Want to Be Like Anzu?

fotc

March 18, 2015

Abstract

Anzu has long been established as awesome. This is reinforced by her performance in a recent game show on which she performed mathematical computations with surprising rapidity. We will show that it is humanly possible to perform such computations mentally with similar precision in a similar time frame using common approximations and equipped with a minimum of physics and mathematics knowledge. In doing so, we will show that it is possible to be like Anzu. And thus Miku is best Cinderella Girl.

1 Introduction

Anzu is awesome (Fig. 1). She is not nearly as awesome as Miku (Fig. 2), but Anzu is undeniably awesome.

Anzu was able to solve a physics problem on a game show with lightning speed. If we are able to duplicate similar results, we will also be like Anzu. To do so, we will attempt the same problem using a common approach in physics and engineering based on approximation and error correction.

1.1 The Problem

Suppose you dropped an apple from the top of the Tokyo Skytree. What will be the velocity of the apple at the instant before it hits the ground? The height of the Tokyo Skytree is 634 meters. Assume that gravitational acceleration is a constant is 9.8 meters per second squared.

1.2 The Solution

Anzu arrived at an answer of 111.474 m/s two seconds after the problem was fully read. The measured time between problem display on the monitor and Anzu's answer is 13 seconds. The upper bound on time to answer to be like Anzu is thus 13 seconds.

2 Being Like Anzu

2.1 Step 1: Kinematics

The first step is to develop a model for the system. The weather conditions and the dimensions and geometry of the apple are not given, so we will assume that

Figure 1: Anzu is awesome.

Figure 2: Miku is most awesome.

air resistance is negligible or irrelevant. Gravity is given as a constant. Thus, we may reasonably assume a model of simple linear motion under constant acceleration.

Recall the basic kinematic equations for linear motion under constant acceleration from introductory physics.

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{1}$$

$$v = v_0 + at \tag{2}$$

$$v^2 = v_0^2 + 2a(x - x_0) \tag{3}$$

(1) relates position to velocity and acceleration. (2) relates velocity to acceleration and time. (3) relates velocity to position and acceleration.

The problem is to find v given x and a; thus, the appropriate equation to use is (3). Assuming that the apple in the problem is dropped from rest and defining positive position and velocity as towards the Earth from the top of the Tokyo Skytree,

$$v = \sqrt{2ax} = \sqrt{2}\sqrt{ax} \tag{4}$$

2.2 Step 2: Approximations

From (4), we see that the relevant quantities are $\sqrt{2}$, $a = 9.8$ m/s, and $x = 634$ m. From these quantities, choose the following approximations

$$\sqrt{2} = 1.41421356237... \approx 1.4 \tag{5}$$

$$a = 9.8 \approx 10 \tag{6}$$

$$x = 634 \approx 640 \tag{7}$$

where we have chosen (7) because we note that we seek to find $\sqrt{ax}$. Using these quantities, we find that

$$\hat{v} = 1.4\sqrt{10 \times 640} = 1.4\sqrt{6400} = 1.4 \times 80 = 112 \tag{8}$$

The difference between this value and Anzu's answer is 0.526 m/s, or a relative error of 0.472%. The author arrived at this answer four seconds after the problem was displayed on the monitor.

2.3 Step 3: Error Estimation

We will use a relative error method to estimate the error induced by our approximations. The only operations in (4) are multiplications and the square root, which greatly simplify error estimation. Relative errors add when their related quantities are multiplied, and relative errors are halved when acted upon by a square root. This can be shown using a first-order Taylor approximation (linearized error) because the expected error is small.

Error (%)	Half Error (%)	Square Root Error (%)	Overadjustment (%)
1.000	0.500	0.499	0.001
2.000	1.000	0.995	0.005
3.000	1.500	1.489	0.011
4.000	2.000	1.980	0.020
5.000	2.500	2.470	0.030

Table 1: The effect of the square root function on small errors.

- The approximation used for $\sqrt{2} = 1.41421356237... \approx 1.414$ was 1.4. This is slightly more than 1% less than the true value.
- The approximation used for $a = 9.8$ was 10. This is slightly more than 2% more than the true value.
- The approximation used for $x = 634$ was 640. This is slightly less than 1% more than the true value.

Thus, we find that the first-order error can be approximated as

$$\varepsilon \approx \underbrace{\frac{1}{2}}_{\sqrt{\cdot}}(\underbrace{0.02}_{a} + \underbrace{0.01}_{x}) - \underbrace{0.01}_{\sqrt{2}} = 0.015 - 0.01 = 0.005 \tag{9}$$

or a 0.5% error, which is close to the 0.472% error calculated in the previous section. Since we expect the approximation to be 0.5% more than the actual answer, and 0.5% is a small amount compared to the originally calculated value, we can subtract 0.5% of the originally calculated value. We thus find that

$$\hat{v}' = (1 - \varepsilon)\hat{v} = \hat{v} - \varepsilon\hat{v} = 112 - 0.56 = 111.44 \tag{10}$$

The difference between this value and Anzu's answer is 0.034 m/s, or a relative error of 0.0305%. The author arrived at this answer seven seconds after the problem was displayed on the monitor.

2.4 Step 4: Error Correction

Since this is not sufficient precision for our purposes, we must correct the error in the estimated error.

- In applying the estimated error incurred by approximating $\sqrt{2}$, we have accounted for about 1.414 of the true value. This leaves a small portion unaccounted for, approximately 0.015% of the true value. We will need to add another 0.015% to account for this.
- In the approximation used for $\sqrt{ax}$, we assumed that error was simply halved. In reality, it is slightly more than halved, following Table 1. We had 3% error before the square root, so we have overadjusted by 0.011%. We will need to add 0.011% to account for this.

Based on this, we see that we have overestimated the error by approximately 0.026%, so the error in $\hat{v}$ was actually closer to 0.474%—which is very close to the true error of 0.472%. In the previous section, we found that 0.5% was approximately 0.56, so 0.026% is approximately one-twentieth plus one-five-hundredth of 0.56, or 0.028 plus 0.001. With this adjustment, we find

$$\hat{v}'' = \hat{v}' + 0.028 + 0.001 = 111.473 \tag{11}$$

The difference between this value and Anzu's answer is 0.001 m/s, or a relative error of 0.0009%. The author arrived at this answer thirteen seconds after the problem was displayed on the monitor.

A slightly more accurate answer can be arrived at in about 10 more seconds of mental computation; however, the scope of the experiment was only to determine how close to the awesomeness of Anzu the author could be. The author has proved that the author is close to being as awesome as Anzu, and because Anzu is so awesome, the difference is negligible. Thus the author is as awesome as Anzu.

3 Conclusion

Anzu is awesome. However, her feats are humanly possible. And since the author is awesome like Anzu, and the author says that Miku is best Cinderella Girl, then it must be true that Miku is best Cinderella Girl.